Radicals, roots, and rational exponents define the multiples or divisors of an expression as a multiple of 1.

- An exponent of 0 indicates that the expression is always equal to 1.
- The exponent 1 indicates that the expression is singular.
- Any integer exponent greater than 0 indicates the number of times the expression multiplies itself.
- Any integer exponent less than 0 indicates the number of times the expression divides itself (as a multiple of 1).

\[10,000 = 10^1 \times 10^1 \times 10^1 \times 10^1 \text{ or } 10^0 = 10^1 + 1 + 1 + 0 = 10^4 \text{ or } (10)^4\]

\[1,000 = 10^1 \times 10^1 \times 10^1 \times 1 = 10^3\]

\[100 = 10^1 \times 10^1 \times 1 = 10^2\]

\[10 = 10^1 \times 1 = 10^1\]

\[1 = 10^0 \times 1 = 10^0\]

Any integer exponent less than 0 indicates the number of times the expression divides itself (as a multiple of 1).

\[.10 = \frac{1}{10^1} = 10^{-1}\]

\[.01 = \frac{1}{10^2} = 10^{-2}\]

Exponents do not have to be integers:

\[10^{4.5} = 10^1 \times 10^1 \times 10^1 \times 10^{0.5} \approx 31,623.\]

Since \(4.5 = \frac{9}{2}\), \(10^{4.5}\) can be rewritten as \(10^{\frac{9}{2}}\). In its radical form, the number would look like this: \(\sqrt[2]{10^9}\).

Here, 2 is the index of the radical, and 9 is the exponent of the radicand 10.

Radicals allow us to work with fractions, which are more accurate representations of numbers than decimals, which are limited to expressing approximations of base 10. If you see an expression in radical form, you can convert it to exponential form by converting the index of the radical to the denominator of the exponent and the exponent of the radicand to the numerator in the new exponent.

In the expression \(\sqrt[2]{10^9}\), the radicand (that which is being multiplied, divided, or both by itself a certain number of times) happens to be 10, the base of our decimal system. Of course, you can use other radicands as bases. Consider \(\sqrt[2]{16} \div 81\), which can also be written as \(\sqrt[2]{16} \div \sqrt[2]{81}\) or \(\left(\frac{16}{81}\right)^{\frac{1}{2}}\).

If you rewrite 16 as \(4^2\) and 81 as \(9^2\), the expression becomes \(\sqrt[2]{4^2} \div \left(\frac{4^2}{9^2}\right)^{\frac{1}{2}} = \left(\frac{4}{9}\right)^{\frac{1}{2}} = \frac{4}{9}\).
Simplify the following: \( \sqrt[3]{64} = \sqrt[3]{8^2} = 8 \)
\( \sqrt[3]{8x^6} = \sqrt[3]{2^3 \cdot x^6} = 2x^2 \)
\( \sqrt[4]{256(x+1)^4} = \sqrt[4]{4^4 (x+1)^4} = 4(x+1) \)
\( 8^{\frac{1}{3}} = 2 \)
\( \left(-27x^6y^7\right)^{\frac{1}{3}} = (-3x^2y^7)^{\frac{1}{3}} = -3x^\frac{2}{3}y^{\frac{7}{3}} \)